LAB 9: Frequency Response

1. Objective

This week, students will experimentally investigate the magnitude and phase frequency response of several circuits. We will begin by studying a speaker "cross-over" circuit. Such a circuit directs the low frequency content of a music signal to a "woofer" (a large cone speaker designed to reproduce low frequency acoustic signals accurately). The midrange frequencies are directed to a "midrange" speaker and the high frequency content is directed to a "tweeter" (designed to reproduce high audio frequencies). Next we will examine a frequency selective filter called a "band-stop" or "notch" filter. This RLC configuration removes a narrow band of frequencies from a signal while leaving the other frequencies virtually unaltered.

2. Background

<u>2.1 What is frequency response?</u>

We know from our discussions about phasors, that a sinusoidal source in an RLC circuit produces sinusoidal voltages and currents throughout the circuit at the same frequency. The only difference is in the peak values (magnitudes) and phases of the sinusoids. That is, if the input voltage is known to be

$$v_{in}(t) = V_{in}\cos(wt + f_{in}) \ , \qquad (1)$$

then a voltage elsewhere in the circuit (which can be declared the output) must be of the form

$$v_{out}(t) = V_{out}\cos(wt + f_{out}) \ . \qquad (2)$$

In the phasor domain, we have

$$\mathbf{V}_{in} = V_{in}e^{jf_{in}} \ , \qquad (3)$$

and

$$\mathbf{V}_{out} = V_{out}e^{jf_{out}} \ . \qquad (4)$$

We can analyze a particular circuit in the phasor domain with this "generic" sinusoidal/phasor input. That is, we let the input be of the form in (1) in the time domain and (3) in the phasor domain, without specifically specifying the amplitude, frequency or phase. In the analysis, the phasor output can be written in terms of the "generic" phasor input. Doing so, one will see that the output is the input scaled by some complex number. This complex number will usually depend on the input voltage frequency. This gives us

$$\mathbf{V}_{out} = \mathbf{V}_{in}H(w) \ . \qquad (5)$$

The scaling parameter, $H(w)$, which generally depends on the input frequency, is called the circuit's frequency response. This function will differ from circuit to circuit, but the form in (5) applies to any linear circuit (one made of R's L's and C's). Frequency response tells us how any sinusoidal input is altered by the system.

Because inductors and capacitors have frequency-dependent complex-valued impedances, any circuit containing inductors or capacitors will have a frequency-dependent complex-valued frequency response. Note that $H(w)$ can be written in polar form as

$$H(w) = |H(w)|e^{j\angle H(w)}, \tag{6}$$

where $|H(w)|$ is the magnitude frequency response and $\angle H(w)$ is the phase frequency response. To visualize this complex valued function, we must use two separate plots, $|H(w)|$ vs. w, and $\angle H(w)$ vs. w.

Rewriting (5) in polar form yields

$$V_{out}e^{jf_{out}} = |H(w)|e^{j\angle H(w)}V_{in}e^{jf_{in}} = |H(w)|V_{in}e^{j(\angle H(w)+f_{in})}. \tag{7}$$

By performing an inverse phasor transform, the output voltage in the time domain can be expressed as

$$v_{out}(t) = |H(w)|V_{in}\cos(wt + \angle H(w) + f_{in}). \tag{8}$$

By inspecting frequency response plots for a given circuit, you can quickly gain insight into how the circuit will respond to any sinusoidal input. In particular, for a single sinusoidal input, the output amplitude is the input amplitude **multiplied** by the magnitude frequency response evaluated at the frequency of the input. The output phase is the input phase **plus** the phase frequency response evaluated at the frequency of the input.

Furthermore, by keeping in mind that most any waveform can be expressed as a superposition of sinusoids, then knowing how your circuit responds to various sinusoids gives you insight into how the circuit will respond to **ANY** input! For example, a music signal can be thought of as having many sinusoidal components at a wide range of audible frequencies (20Hz-20kHz). If the magnitude frequency response for a specific circuit drops off at higher frequencies (making it a low-pass filter), this tells us that that circuit will quiet the high frequency components. This, in turn, yields a smoother signal when viewed on an oscilloscope and a muffled sounding signal when converted back into an acoustic signal. This is the same effect you get when you turn down the treble on your stereo.

Speaking of stereo equipment, consider this. When you set a graphic equalizer on a stereo, the shape of the curve you form with the slider knobs defines the magnitude frequency response of the equalizer system. The idea here is that you can compensate for the acoustic frequency response of the room and surroundings (which is why it is called an "equalizer"). For example, if you have plush furniture and carpet, the room may tend to absorb high frequency energy (rather than reflecting it into your ear). This means that the room is acting as an acoustic low-pass filter. In such a case you may need to boost the high frequency content to "equalize" this affect. Thus, we set the frequency response of the equalizer to act as a high pass filter (boosting the treble). Ideally, we want the frequency response of the equalizer times the frequency response of the room to be a constant with respect to frequency (flat frequency response). This way, you will hear what the musician had intended for you to hear.

2.2 How do I measure a system's frequency response?

When determining any system's frequency response, you must excite the system (in our case a circuit) with a sinusoidal signal over a range of frequencies (a frequency sweep) and observe the relationship between the input and output at each frequency. From Equation (5), we can solve for the frequency response as follows

$$H(w) = |H(w)| e^{j\angle H(w)} = \frac{\mathbf{V}_{out}}{\mathbf{V}_{in}} = \frac{V_{out} e^{jf_{out}}}{V_{in} e^{jf_{in}}} = \frac{V_{out}}{V_{in}} e^{j(f_{out} - f_{in})}. \qquad (9)$$

Thus, the magnitude frequency response is given by

$$|H(w)| = \frac{V_{out}}{V_{in}} \qquad (10)$$

and the phase frequency response is given by

$$\angle H(w) = f_{out} - f_{in}, \qquad (11)$$

where w is the frequency of the input sinusoid in radians/second. By making many measurements of these quantities at different frequencies, we can generate magnitude and phase frequency response plots.

2.3 How does a speaker work and what is a cross-over circuit?

In the early 1800's, Michael Faraday observed that when current flows through a wire, a magnetic field is created that can be observed by placing iron filings nearby and watching them align with the magnetic field (like they do when placed around a fixed magnet). Furthermore, Faraday determined that when a magnet is moved near a wire, an impetus for charge to move is observed (i.e., a voltage, the driving force for current). The same affect is observed if the wire moves relative to a fixed magnet. His work brought about the widespread understanding that electric and magnet forces are inexorably linked. Faraday's insightful observations have led to inductors, transformers, electric motors and generators, relays, as well as a variety of transducers such as speakers and microphones. These transducers convert electrical signals into acoustic signals and vice versa.

A speaker can be made by attaching a paper cone to an electromagnet. This electromagnet is simply a coil of wire wrapped around an iron core. The electromagnet is set inside a fixed magnet. When current flows through the electromagnet coil, it becomes a magnet which wants to move inside the fixed magnet, in order to satisfy its natural tendency to align its North-pole to the fix magnet's South-pole. The amount of movement is proportional to the current. This gives us a way to convert an electrical signal into mechanical movement, which creates an acoustic signal. By the way, a speaker can be used as a microphone by operating it in reverse. Talking into a speaker causes the cone to move, which moves the wires of the electromagnet with respect to the fixed magnet. This induces a voltage. It is not an efficient microphone, because the cone requires a strong acoustic signal to make it move, but it does work.

By observing the voltage-current relationship for a typical speaker, you can see that its impedance is approximately $8\,\Omega$ (Although some are $2\,\Omega$ or $4\,\Omega$). Careful inspection may reveal that there is a slight phase shift between voltage and current, indicating that a more precise model should involve a complex valued impedance (one that affects phase). However, in many cases, it will suffice to use the simple resistor model.

One problem with a speaker is that a large slow moving cone is needed to reproduce low frequencies effectively, while a small fast moving cone is required to reproduce high frequencies. In order to reproduce a wide range of frequency components effectively, a speaker often uses multiple cones in one housing. We will consider a speaker with three cones: a "woofer," designed to reproduce low frequency acoustic signals accurately; a "midrange" speaker for middle frequencies; and a "tweeter" for the high frequency content. What we need to do is make sure that the woofer only sees low frequency electrical signals, so that it does not attempt to move more quickly than it is capable of (which would lead to distortion). Likewise, we want the tweeter to only see high frequency electrical signals so that it does not attempt to reproduce low frequency acoustic signals, which it is incapable of. The midrange speaker should only see midrange frequency electrical signals. In order to direct the various frequency components to the appropriate cone, we use a cross-over circuit. A cross-over circuit is basically a low-pass filter, a band-pass filter, and a high-pass filter working to separate the music signal into three frequency ranges.

3. Procedure

3.1 Cross-over circuit

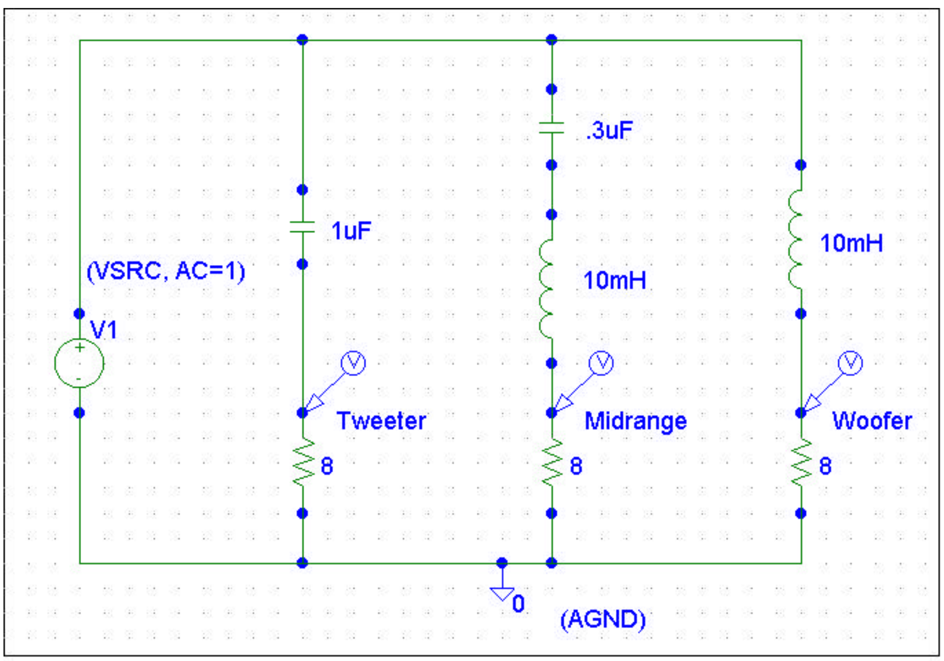

Figure 1: Cross-over circuit.

A cross-over circuit is shown in Figure 1. Note that the woofer, midrange, and tweeter are modeled as 8Ω resistors. Begin by constructing the tweeter circuit using the speaker provided where the 8Ω resistor is shown on the schematic. Let the input be a sinusoidal voltage with a peak-to-peak amplitude of 1V. **Keep the amplitude of the voltage source small so that we keep the sound at a safe level!** Monitor the source voltage on Channel 1 and the voltage across the speaker on Channel 2. Be aware that since the waveform generator has a finite source resistance, the measured voltage on the oscilloscope may be smaller than the settings indicate on the waveform generator. This is noticeable when the impedance of the load is small, making the voltage divider effect more pronounced.

(1) Sweep the frequency of the source and observe the input and output amplitudes on the oscilloscope. Determine the basic frequency response type (low-pass, high-pass, band-pass, or notch).

(2) Measure the magnitude and phase frequency response at these frequencies: 20, 40, 60, 80, 100, 200, 400, 600, 800, 1000, 2000, 4000, 6000, 8000, 10000, 20000Hz (and more if you want). You may replace the speaker with an 8Ω resistor, if you want to keep the noise down. The same frequency response should be observed.

(3) Calculate the theoretical frequency response and use MATLAB to evaluate your expression and plot it along with your experimental data for comparison. You should have one figure showing the magnitude frequency response vs. frequency. This figure should have two curves, theoretical and experimental. You should have a second figure showing the phase frequency response vs. frequency. This figure should also have two curves, theoretical and experimental. Use a legend to identify the two curves and be sure to label axes (including units). Plot the curves versus frequency in Hz on a log axis. **See Appendix A**.

Construct the midrange circuit using the same speaker.

(4) Sweep the frequency of the source and observe the input and output amplitudes. Determine the basic type of frequency response (low-pass, high-pass, band-pass or notch).

(5) Measure the magnitude and phase frequency response at these frequencies: 20, 40, 60, 80, 100, 200, 400, 600, 800, 1000, 2000, 4000, 6000, 8000, 10000, 20000Hz (here you will **need** additional measurements in the pass-band region). Again, you may replace the speaker with an 8Ω resistor if you want.

(6) Calculate the theoretical frequency response and use MATLAB to evaluate your expression and plot it along with your experimental data for comparison. You should generate two figures as done in (3); one for the magnitude frequency response and one for the phase frequency response, each containing both theoretical and experimental curves.

Construct the woofer circuit using the same speaker.

(7) Sweep the frequency of the source and observe the input and output amplitudes. Determine the basic type of frequency response (low-pass, high-pass, band-pass or notch).

(8) Measure the magnitude and phase frequency response at these frequencies: 20, 40, 60, 80, 100, 200, 400, 600, 800, 1000, 2000, 4000, 6000, 8000, 10000, 20000Hz (and more if you want). Again, you may replace the speaker with an 8Ω resistor.

(9) Calculate the theoretical frequency response and use MATLAB to evaluate your expression and plot it along with your experimental data for comparison. You should generate two figures as done in (3); one for the magnitude frequency response and one for the phase frequency response, each containing both theoretical and experimental curves.

(10) The cross-over circuit component values were selected for convenience. Do you see any problems with the frequency responses in this cross-over circuit? If so, how would you change the shape of the frequency responses to improve the design?

3.2 Band-stop (Notch) filter

A band-stop filter is designed to eliminate a specific range of input frequencies. This can be helpful, for example, to remove an undesired hum or tone from an audio signal. Note that audio electronics are typically powered by a 60Hz AC outlet. The AC is converted to DC, but not perfectly, leaving a 60Hz ripple. This ripple in the power source tends to create a slight hum at 60Hz in some systems. A notch filter, with a notch frequency of 60Hz, can be used to remove this hum prior to the signal reaching the speakers. However, be aware that when the music signal has a component at 60Hz, that too will be lost.

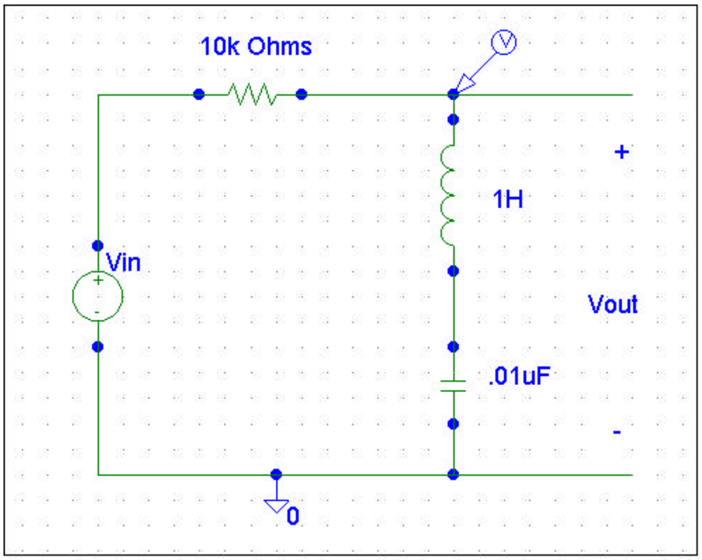

Figure 2: Series RLC circuit configured as a band-stop filter.

Begin by constructing the circuit shown in Figure 2. Let the input be a 1V peak-to-peak sinusoid. Monitor the input on Channel 1 and the output on Channel 2. You may recall that this series RLC circuit is one we have seen before and it has a resonant frequency of $w_0 = 1/\sqrt{LC}$. When we define the output as shown in Figure 2, the circuit becomes a notch filter and the resonant frequency translates into the notch frequency. Note that the impedance of the L and C series combination goes to zero at the resonant frequency. That is,

$$jwL + \frac{-j}{wC} = 0 \tag{12}$$

at $w = 1/\sqrt{LC}$. Thus, using a voltage divider analysis, it is clear that the output will go to zero at that frequency.

(11) Measure the magnitude and phase frequency response at these frequencies: 20, 40, 60, 80, 100, 200, 400, 600, 800, 1000, 2000, 4000, 6000, 8000, 10000, 20000Hz, plus **several** additional frequencies near the band-stop region (to later create an accurate plot).

(12) Calculate the theoretical frequency response for the notch filter and use MATLAB to evaluate your expression and plot it along with your experimental data for comparison. You should generate two figures as done in (3); one for the magnitude frequency response and one for the phase frequency response, each containing both theoretical and experimental curves.

(13) Use PSPICE to simulate the notch filter circuit in a frequency sweep analysis mode. Draw the circuit in PSPICE. Set the voltage source (VSRC) parameter to AC=1. Use a PSPICE voltage marker at the positive output node, as shown in Figure 2. Under set-up, select AC sweep. Set the frequency sweep parameters for decade sweep using start frequency=20, end frequency=20000, and points per decade=101.

(14) What component values should be chosen to create a notch filter at 60Hz?

APPENDIX A: MATLAB Example

$$H(w) = \frac{1}{1 + jwRC}$$

```
f=[20,40,60,80,100,200,400,600,800,1000,2000,4000,6000,8000,10000,20000];
w=2*pi*f;

R=1000;
C=1e-6;
H=1./(1+j*w*R*C);

figure(1)
semilogx(f,abs(H),'ro-');
legend('Theoretical');
xlabel('f (Hz)');
ylabel('|H(f)|');
title('Magnitude Frequency Response')
axis tight

figure(2)
semilogx(f,angle(H)*180/pi,'ro-');
legend('Theoretical');
xlabel('f (Hz)');
ylabel('\angle H(f) (degrees)');
title('Phase Frequency Response')
axis tight
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